STRENGTH OF MATERIALS

J. K. WHILDIN

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MEMORANDA

ON THE

STRENGTH OF MATERIALS

USED IN

ENGINEERING CONSTRUCTION.

COMPILED AND EDITED BY

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PREFACE.

MANY valuable accessions to our knowledge of the strength of materials have been made within a comparatively recent period; the records, however, have been scattered through different books, periodicals, &c. not always accessible or convenient for reference. We have therefore endeavored in this brief essay to bring together and arrange some of the more important elements, both old and new, so as to form an "*aide memoire*" for the Engineer and Architect, at the drawing office and elsewhere.

The sources of information have been given as far as possible, in order that the results may be checked, and opportunities afforded for lengthened investigations, which are beyond the scope of these papers.

Wherever it has been necessary to express the strength of any material in tons, we have modified the tables and formulas to admit the use of the American ton of 2,000 lbs. as promotive of simplicity.

CONTENTS.

TENSION.

Tensile strength of timber.
do do metals.
do do stones.
do do wire rope and hempen cable.
Strength of thin cylinders of cast iron.
Modulus of elasticity.
Strength of thick cylinders, as cannon, &c.
Effects of re-heating, &c.

COMPRESSION.

Resistance of timber to crushing.
do metals do
do stone do
Experiments on brickwork, &c,
Strength of pillars.
Collapse of tubes.

DETRUSION.

Experiments on punching and shearing.

TRANSVERSE STRENGTH OF MATERIALS.

Beams of uniform strength.
Table of co-efficients of timber, stone and iron.
Relative strength of wrought and cast iron.
Transverse strength of alloys.
Table of experiments on cast iron beams of various sections.
do do wrought iron do do
Edwin Clark's co-efficients for wrought and cast iron beams.

LATTICE GIRDERS.

TRUSSED CAST IRON GIRDERS.

DEFLECTION OF BEAMS.

TORSIONAL STRENGTH.

TORSIONAL ELASTICITY.

TABLE

Showing the Tensile strength of Timber.

Description.	Weight necesary to tear asunder one square inch.		Modulus of elasticity in lbs.	Authority.
	In pounds Avoirdupois.	In tons of 2,000 lbs,		
Timber and other organic fibre.				
Ash, (English)	17,000	8.5	1,600,000	Barlow.
Bamboo	6,300	3.6		Rankine's Mechanics.
Beech, (English)	11,500	5.7	1,350,000	Barlow.
Birch "	15,000	7.5	1,645,000	Rankine's Mech.
Box	20,000	10.		Barlow.
Cedar	11,400	5.7	486,000	Bevan.
Chestnut, from	10,000	5.0	1,140,000	Rankine's Mech.
Chestnut, to	13,000	6.5		
Cyprus	6,000	3.		
Deal { Christiana	11,000	5.5		Barlow.
Deal { Memel	11,000			
Elm	5,780	2.8		Do.
Fir, or red Pine (New England)	12,000	6.		Do.
Do. common American	8,800	4.4		
Hawthorn	10,500	5.2		Bevan.
Hazel	18,000	9.		Do.
Hempen cables	5,600	2.8		Rankine's Mech.
Holly	16,000	8.		Bevan.
Hornbeam (dry)	20,000	10.		Do.
Lancewood	23,400	11.7		
Larch (Scotch)	7,000	3.5		Barlow.
Lignum vitae	11,800	5.9		Mushenbroek.
Locust	20,580	10.2		Barlow.
Do. (Robinia Pseudo Acacia)	16,000	8.		
Mahogany	8,000	4.		
Mahogany	21,000	10.5	1,255,000	
Maple (Norway)	10,584	5.2		Moseley's Mechanics.
Mulberry	10,600	5.3		
Mulberry	17,400	8.7		
Oak (American white)	11,501	5.7		Barlow.
" (English)	9,000	4.5	1,200.000	Do.
" (English)	15,000	7.5	1,750,000	
" (African)	14,000	7.		Do.
" (Adriatic)	14,000	7.		Do.
" (Canadian)	12,000	6.		Do
" (Dantzic)	14,500	7.2		Do.
Pear	9,800	4.9		Do.
Pine, Pitch	10,500	5.2		Do.
Plane tree	11,700	5.8		Bevan.
Plum tree	11.300	5.6		Hodgkinson.
Poplar	7,200	3.6	1,130,000	Weisbach's Mechanics.
Quince	5,800	2.9		
Quince	8 800	4.4		
Sycamore	13 000	6 5	1 040,000	Bevan.
Teak	15 000	7.5	2,400,000	
Walnut	8,130	4 06		Mushenbroek.
Whalebone	7,667	3.8	820,000	Moseley's Mech.
Willow, dry	14.000	7.		Bevan.
Yew, Spanish	8.000	4.		Do.
Ivory	16,626	8.3		Moseley's Mech.

TENSILE STRENGTH OF METALS.

Name of Material.	Weight necessary to tear asunder one square inch. In pounds Avoirdupois.	In tons of 2,000 lb.	Modulus of elasticity in lbs.	Authority.
Antimony cast	1,060	0.5		Muchenbrock.
Bismuth, do.	3 250	1.6		Do.
Brass, fine yellow, cast	17,968	8.9	8 930,000	Rennie.
Do. Wire	49,000	24.5		
Copper, cast.	19,000	9 5		Rennie.
Do. Sheets	30,000	15.		Franklin Institute.
Do. Bolts	33 000	16.5		Rennie.
Do. Wire	60 000	30.	17,000,000	Rankine's Mech.
Gun Metal (Copper and Tin)	36,000	18.		Rennie.
Lead, cast (English)	1,824	0.9		Do.
Do. milled or sheet	3.300	1.6	720,000	Tredgold.
Steel	100,000	50.	29,000,000	Rankine's Mech.
	130.000	65.	42,000,000	Do. do.
Tin, cast	4,600	2.3	4,000,000	Rennie
Do. do.	5,322	2.6		Mushenbroek.
Platinum Wire	56,000	28.		U. S. Ordnance Manual 1850.
Silver	40,000	20.		
Gold, cast	20,000	10.		Mushenbroek.
Do. Wire	30,000	15.		Ann. de Chimie XXV
Zinc	7.000	3.5	13,000,000	Rankine's Mech.
	8,000	4.		

TENSILE STRENGTH OF IRON.

Cast Iron	20,834	10.4		Franklin Institute.
	19,200	9 6		Rennie.
	27,700	13.8		Cubitt.
Carron No. 2, cold blast	16.683	8.3	17,270,500	Experiments by E. Hodgkinson, for the "Railway Commission" (England) 1849.
Do. hot blast	13,505	6.7	16,085,000	
Do. No 3, cold blast	14,200	7.1	16.246 000	
Do. hot blast	17 755	8.8	17 873,000	
Devon No 3, hot blast	29,107	14.5	22,473,000	
Buffery No. 1, cold blast	17,466	8.7	15 381,000	
Do. hot blast	13,434	6.7	13,730,000	
Coed No 2, cold blast	18 855	9.4	14,313 000	
Do. hot blast	16.676	8.3	14,322,000	
Common Pig Iron	15,000	7.5		Experiments by Maj. Wade for the U. S. Ordnance department.
Good common castings	20.000	10.		
Gun heads, specimens from	24.000	12.		
	39 500	19.7		
Sterling's toughened Iron	28,000	14.		E. Hodgkinson.

Wrought Iron.

Bar Iron.	Salisbury, Conn.	66 000	33.	29,000,000	Franklin Institute.
	Bellefonte, Pa.	58.000	29.		
	English	56,000	28.		
	Pittsfield, Mass.	57,000	28.5		Major Wade.
	Maramec, Mo.	43,000	21.5		
		53,000	26.5		

Tensile strength of Iron.

Name of Material.	Weight necessary to tear asunder one square inch. In pounds Avoirdupois.	Weight necessary to tear asunder one square inch. In tons of 2,000 lbs.	Modulus of elasticity.	Authority.
WROUGHT IRON.				
Chain Iron	43,000	21.5		C. H. Haswell, Wash. Navy Yard.
English rivet Iron	61.000 70.000	30 5 35.		W. Fairbairn.
Average Boiler plate	51,000	25 5		Rankin's Mech. 1858.
Do. joints double riveted	35,700	17.8		
Do. do. single do.	28,600	14.3		
Russian Iron	62 640	31.3		United States Board of Ordnance.
English rolled iron	56 530	28.2		
Lowmoor do.	56,100	28.		
American hammered	53,900	26.9		
Cast Steel (highest)	142 200	71.1		Mallet.
Do. (mean)	88,600 134,000	44.3 67.		
Do. tempered	150,000	75.		
Shear Steel	124.000	62.		
Blister Do.	133,000	66.5		
Puddled Steel, average of three samples	112,000	56.		Mersey Steel and Iron Co., England, 1857.

Natural and Artificial Stones.

Brick	200			Rankine's Mech.
Cement	300			
Glass	9,400		8,000,000	
Slate	9 600 12,800		13 000,000 16,000,000	
Mortar Hydraulic	100 to 170			Vicat.
Do. common	10 to 50			Do.

Strength of Iron Wire Rope and Hempen Cable,

BY J. A. ROEBLING, C. E.

	Circumference of Wire Rope in inches.	Trade Number.	Circumference of Hemp Rope of equal strength in inches.	Breaking weight in tons of 2,000 lbs.
Fine Wire	6 62	1	15½	74.
	6.20	2	14½	65.
	5.44	3	13	54.
	4.90	4	12	43.6
	4.50	5	10¾	35.
	3 91	6	9½	27.2
	3.36	7	8	20.2
	2.98	8	7	16.
	2.56	9	6	11.4
	2.45	10	5	8.64
Coarse Wire	4.45	11	10¾	36.
	4.00	12	10	30.
	3 63	13	9½	25.
	3.26	14	8¼	20.
	2.98	15	7¼	16.
	2.68	16	6¼	12.3
	2.40	17	5½	8.8
	2.12	18	5	7.6
	1.9	19	4.75	5.8
	1.63	20	4	4.09
	1.53	21	3.3	2.83
	1.31	22	2.80	2.13
	1.23	23	2.46	1.65
	1.11	24	2.2	1.38
	0.94	25	2 04	1.03
	0.88	26	1.75	0.81
	0.78	27	1.50	0.56

Practical Rule for finding the Thickness of Cast Iron Pipes and other Thin Cylinders, which are exposed to Internal Pressure.

An Extension of the Rule given by J. C. TRAUTWINE, C. E., for water pipes only.

"Most authors who allude to this subject content themselves with merely giving theoretical rules, which are well known in practice to furnish results entirely too low. Thus Barlow's rule gives for a 16 inch water pipe, to support a head of 100 feet, a thickness of but $\frac{1}{100}$ of an inch, or about twice that of ordinary letter paper. Other writers again give the results of some very incomplete experiments, altogether too limited in number to serve as general data, while others either pass over the subject in silence, or at most with an intimation that it admits of no specific instructions.

* * * Mr. Barlow, in giving his rule, assumes the *safe* cohesive strength of cast iron at 18,000 lbs. per square inch; but Mr. Hodgkinson has since conclusively shown, by numerous experiments, that it is not safe to employ more than 15,000 lbs. per square inch as the *ultimate* cohesive strength of ordinary cast iron. I have assumed 5,000 lbs. per square inch as the safe limit for water pipes abstractly considered."

Mr. Trautwine then proceeds to show that a certain thickness is necessary to ensure sound casting, and subsequently to bear handling. Now, it is evident that these same practical considerations apply to all other cast iron cylinders; those for gas and steam included.

Hence, with the foregoing data, we furnish the following:—

Let a=The thickness necessary to ensure good casting, &c., varying according to the values given on next page for pipes of different diameters.

Let P = The pressure in lbs. per square inch on the inner surface of pipe.

Let D = The inside diameter of pipe in inches.

Let T = The entire thickness of pipe in inches.

Then $T = a + \frac{DP}{10000}$ - - - - - No. 1

Values of a—

$a =$

For pipes less than 12 diam. - - - - -	$\frac{3}{8}''$
" from 12 to 30 " - - - - -	$\frac{1}{2}''$
" " 30 " 48 " - - - - -	$\frac{5}{8}''$
" " 48 " 70 " - - - - -	$\frac{3}{4}''$
" " 70 " 100 " - - - - -	$\frac{7}{8}''$

Example.—What must be the thickness of the cylinder of a high pressure engine, which is 31 inches bore and exposed to a pressure of 100 lbs. per square inch?

Referring to the table we find $a = \frac{5}{8}$; also, we know $D = 31$, and $P = 100$.

Hence, $T = a + \frac{DP}{10000} = \frac{5}{8} + \frac{31 \times 100}{10000} = \frac{5}{8} + .31 = \frac{15}{16}''$

It is customary in steam cylinders to allow some extra thickness for wearing or reboring, so that the above thickness might be increased to $1\frac{1}{16}$ or $1\frac{1}{8}$.

THE MODULUS OF ELASTICITY,

Or the Resistance of Materials to Stretching.

This is the quotient arising from the division of any weight (W) in lbs, by the relative extension which it produces when suspended from a rod one inch square, or it may be expressed thus:—Extension: length:: weight producing extension: modulus of elasticity; also, representing the entire length of rod by L; the extension, or the additional length produced by the weight, by e; the weight in lbs.,

which produces the extension by W; and the modulus of elasticity in lbs. by M, we have as above, $e : L :: W : M$,

whence $M = \frac{LW}{e}$, and $e = \frac{LW}{M}$, for rods one inch square.

For rods having any other area of cross section, let A represent that area in square inches; then the above formulas become, $M = \frac{LW}{Ae}$ - - - - - - - No. 2

and $e = \frac{LW}{AM}$ - - - - - - - No. 3

Example.—What extension will be produced in a bar of wrought iron 3 inches diameter and 400 inches long, having in suspension a weight of 84,000 lbs.? The average modulus of elasticity of wrought iron may be taken at 28,000,000 lbs.$= M$; also, $A = 7$ square inches, $L = 400$ inches, and $W = 84{,}000$ lbs.; hence,

$$e = \frac{LW}{AM} = \frac{400 \times 84{,}000}{7 \times 28{,}000{,}000} = \frac{33{,}600{,}000}{196{,}000{,}000} = \frac{12}{70} = 0.171 \text{ of an inch.}$$

Extracts from a Paper on the Strength of Cannon and other Thick Cylinders, by Capt. Blakely, R.A.

(See Civil Engineer and Architect's Journal, for February and March, 1859.)

"It has long been known that increasing the thickness of metal in the cylinder of a Bramah press does not increase its strength beyond a certain point. The failure, one after another, of the huge presses used in launching the Leviathan, under a pressure of less than five tons to the inch, must be fresh in the memory of all; yet the thickness of metal in some of these was as much as $7\frac{1}{2}$ inches, the diameter of the

piston or ram being only 10 inches; the tensile force of 15 inches, two thicknesses each 7½ inches, being thus opposed to a pressure on only 10 inches. This small proportion of pressure has always been found sufficient to burst a cylinder.

"The cause of this weakness, which equally affects cannon, is even now but very little known, although it was distinctly pointed out by Prof. Peter Barlow in a paper read to Institution of Civil Engineers many years ago, and published in the first volume of their Transactions. Professor Barlow proves that the outer portions of a cylinder have comparatively little strain transmitted to them, and therefore add little to its strength.

"He bases his argument on the well-known law—that metal subjected to a moderate tensile strain, stretches in the direction of the strain, and very nearly in proportion to it; that is to say, if one ton will stretch a bar of a certain length one inch, two tons will stretch it two inches, or six tons six inches. This is generally known as "Hooke's law," *ut tensio sic vis*, and has been found by experiment to be very nearly correct. Prof. Barlow argues that fluid pressure from within a cylinder must stretch it. That it really does so in a gun or rifle is easily proved by placing a leaden ring round one; if it fit tight before the piece is fired, it will be loose afterwards.

"No known material is capable of exerting a tensile force without stretching. The amount of stretching in many materials is so well known, that it can be used as a measure of the tensile force; this is the case with iron.

"Now, a moment's reflection must convince any one that the outside of a thick cylinder cannot stretch as much as the inside. If it did, the mass of metal or other material must be increased during the strain. Let us consider, as an example, a cylinder the size and thickness of the strong

part of a ten-inch gun, ten inches bore, and ten inches of metal round it; and let us imagine this cylinder to be made of homogeneous india rubber. In this case the inner diameter might easily be stretched to double its size, becoming twenty inches instead of ten. Now, it is evident that the outside circumference and diameter cannot be doubled at the same time, or else the latter must become twice thirty, or sixty inches, which would give a thickness of twenty inches, quadrupling the mass of material—which is impossible.

"A moment's reflection shows that the thickness must diminish as the circumference is increased by pressure from within; for, if the thickness remain 10 inches when the internal diameter has become 20, the external diameter must be 20 plus twice 10, or 40 inches. This could not be, unless we imagine what seems impossible, viz.: that the bulk of the material is considerably enlarged; as each inch in length of the cylinder would now contain 1,200 cylindrical inches (the difference between the squares of 40 and 20, the external and internal diameters), whereas, originally, it only contained 800 inches, the difference between the squares of 30 and 10.

"Yet even if the thickness could remain the same, notwithstanding the increase of circumference, the outside layer could only be strained one-third as much as the inside one, because three times as long. The same elongation which would cause a strain of one ounce or one pound in the larger circumference, would cause a strain of three ounces or three pounds in the shorter one; and the elongation which would but moderately strain the one would break the other.

"This reasoning is equally applicable to the minute extension of iron; the increase of one-tenth of an inch in the outer circumference of a 10-inch gun being possible

without fracturing that part, being an elongation of but 1 in 940; whereas the same extension must crack the inside, as no iron could stand an elongation of 1 in 314. Even on this showing, then, the outside of a thick tube cannot do its share of work; a closer examination, however, must convince us that it cannot do even thus much, for the thickness of the material must diminish as the circumference is increased.

"When the inner diameter of the 10-inch cylinder becomes 20 inches, the thickness must diminish from 10 to 7.32 inches the mass of the material in the cylinder, and therefore its cross section remaining the same. The cross section was originally 800 circular inches—800 being the difference between the squares of 30 inches, the outer diameter, and 10 inches the inner, or 900 minus 100. When stretched, the area of the cross section must continue to be 800 round inches.

"Now the thickness of 7.32 inches gives us an external diameter of twice 7.32 or 14.64 added to 20, the internal diameter; in all 34 64 inches, the square of which is 1,200, subtracting 400; the square of 20 leaves 800 round inches as before. In this case the outside of the cylinder is stretched but 4.64 in 30, about 1 in 7, when the inside is stretched to double its original size.

"If the inner diameter be only stretched to 11 inches, the thickness must diminish from 10 to 9.674 inches; the outer diameter becoming 30.348 inches; the cross section remaining 800 round inches as before, the difference between the squares of 30.348 and 11. Here the outer layer is elongated .348 in 30, or 1 in 86; whereas the inner is extended 1 in 10, showing a strain or an exertion of force $8\frac{1}{2}$ times greater.

"In the minute extension of metals, the disproportion is still more striking. Thus, in cast iron the 10-inch inner

diameter may become $10\frac{1}{100}$, which would extend the outer diameter only from 30 to $30\frac{1}{300}$, the cross section remaining 800 inches, and the thickness diminishing from 10 inches to $9\frac{299}{300}$. (*See Fig.* A.)

Fig. A.

Here the outside would only be stretched $\frac{1}{300}$ in 30, or 1 in 9,000; the inside being stretched $\frac{1}{100}$ in 10, or 1 in 1000, exerting therefore nine times as much force as the outside.

"It is evident that a slight increase of pressure from within would break the inside, while the outside could help but little in restraining the disruptive force.

"If we make equi-distant marks on the end of an india rubber cylinder (*see Fig.* B), and stretch it, we can see plainly how much more the inside is strained than the outside, or even the intermediate parts. The spaces between the marks will become thinner, each space becoming less thin than that outside of it, and the inner space *much* thinner than the others (*see Fig.* C), showing that when the inside is strained almost to breaking, the intermediate parts are doing much less work, and those far removed almost none.

Fig. B.

Fig. C.

"The law deduced by Prof. Peter Barlow, as given below, is that in cylinders of metal the force exerted by different parts varies inversely as the square of the distances of the parts from the axis."

Prof. Peter Barlow's Formula for the Strength of Thick Cylinders.

"In the first place it is obvious that, whatever extension the cylinder or ring may undergo, there will be still in it the same quantity of metal; or, which is the same, the area of the circular ring formed by a section through it, will remain the same, which area is proportional to the difference of the squares of the two diameters.

"Let D be the interior diameter before the pressure is exerted, and $D+d$ its diameter when extended by the pressure. Let also D' be the external diameter before, $D'+d'$ the diameter after the pressure is exerted. Then, from what is stated above, it follows that we shall have

$$D'^2 - D^2 = (D'+d')^2 - (D+d)^2, \text{ or,}$$

$$2D'd' + d'^2 = Dd + d^2, \text{ or,}$$

$$2D'+d' : 2D+d :: d : d';$$

or, since d' and d are very small in comparison with D' and D, this analogy becomes $D' : D :: d : d'$. That is, the extension of the exterior surface is to that of the interior as the interior diameter to the exterior.

"But the resistance is as the extension divided by the length; therefore the resistance of the exterior surface is to that of the interior as $\frac{D}{D'} : \frac{D}{D}$ or as $D^2 : D'^2$ That is, the resistance offered by each successive lamina is inversely as

the square of the diameter, or inversely as the square of its distance from the centre; by means of which law the actual resistance due to any thickness is readily ascertained.

"Let r be the interior radius of any cylinder, p the pressure per square inch on the fluid, t the whole thickness of the metal, and x any variable distance from the interior surface. Let, also, $rp=s$ represent the strain exerted at interior surface; then by the law last illustrated we shall have $(r+x)^2 : r^2 :: s : \frac{r^2 s}{(r+x)^2}$ for the strain at the distance x from the interior surface, and consequently $\int \frac{r^2 s dx}{(r+x)^2} + Con.$ = the sum of all the strains, or the sum of all the resistance. This becomes, when $x=t$, $R=r^2 s\left(\frac{1}{r}-\frac{1}{r+t}\right)=s\frac{rt}{r+t}$ That is, the sum of all the variable resistance due to the whole thickness t, is equal to the resistance that would be due to the thickness $\frac{rt}{r+t}$, acting uniformly with a resistance s or rp."

Application of this Rule for Computing the Proper Thickness of Metal in a Cylindric Hydraulic Press of given Power and Dimensions.

"Let r be the radius of the proposed cylinder, p the pressure per square inch on the fluid, and x the required thickness. Let also c represent the cohesive strength of a square inch of the metal; then, from what has preceded, it appears that the whole strain due to the interior pressure

will be expressed by pr, and that the greatest resistance to which the cylinder can be safely opposed is $c \bowtie \frac{rx}{r+x}$; hence, when the strain and resistance are in equilibrio, we shall have $rp = \frac{rx}{r+x} \times c$, or $pr+px=cx$, whence $x=\frac{pr}{c-p}$, the thickness sought."

Effects of Re-heating Iron, &c.

Major Wade's experiments show a very remarkable increase in the strength of cast iron when re-heated a certain number of times, and maintained in a state of fusion some hours previous to pouring; and in a paper read before the Society of Arts in England, by Wm. Clay, of the Mersey Steel and Iron Works, it was shown that wrought iron also increases in strength with each successive piling, heating and rolling up to a certain number of times, after which it decreases at an equal rate. The following table shows the breaking weight in lbs. per square inch of the successive samples:—

1. Puddled bar	43,904	7. Reheated	59,585
2. Re-heated	52,864	8. "	57,344
3. "	59,585	9. "	57,344
4. "	59,584	10. "	54,105
5. "	57,354	11. "	51,968
6. "	61,824	12. "	43,904

"It will thus be seen that the quality of the iron increased up to No. 6 (the slight difference of No. 5 may perhaps be attributed to the sample being slightly defective), and that from No. 6 the descent was in a similar ratio to the previous increase."

Similar experiments on puddled steel bars gave results showing a similar increase and decrease.

RESISTANCE OF TIMBER TO CRUSHING.

Description.	Crushing Weight, per square inch. In lbs.	Crushing Weight, per square inch. In Tens of 2,000 lbs.	Weight in lbs. of a Cubic Foot.	Authority.
Ash	8,683	4.3	43. 53.	These are the results obtained by Mr. Hodgkinson, from experiments on short cylinders with flat ends. Each cylinder was one inch diameter and two inches high. The least values in this table are for those specimens only moderately dry, and the highest, for those which were well seasoned. It appears from these experiments, that, as a general rule, wet timber does not possess more than half the strength of that which is well seasoned.
Baywood	7,518	3.7	51.3	
Beech	7.733 to 19,363	3.8 to 9.6	53. 43.	
Birch, (American)	3,297 to 11,663	1.6 to 5.8	40.5	
Do. (English)	3.297 to 6,402	1 6 to 3 2	49.	
Cedar	5,674 to 5,863	2 8 to 2 9	56. 47.	
Red deal	5,748 to 6 586	2.8 to 3.2	47.	
White deal	6,781 to 7,293	3.3 to 3.6		
Elder	7.451 to 9,973	3.7 to 4.9	43. 36.7	
Elm, Seasoned	10,331	5.1		
Fir, (Spruce)	6,499 to 6,819	3.2 to 3.4		
Hornbeam	4,533 to 7,289	2.2 to 3 6	47. 50.	
Mahogany	8,198	4.09		
Oak, (Quebec)	4,231 to 5,982	2.1 to 2 9	54 5	
Do. (English)	6,484 to 10.058	3.2 to 5.	58.	
Do. Dantzic, very dry	7,731	3.8		
Pine, Pitch	6.790	3 3	41.	
Do. American Yellow	5.375 to 5 415	2.6 to 2.7	28.8	
Do. Red	5,395 to 7.518	2.6 to 3.7	41.	
Poplar	3,107 to 5,124	1.5 to 2.5	24.	
Sycamore	7.082	3 5	43.	
Teak	12,101	6.	41.	
Larch (Green)	3,201	1.6	32.	
Larch (Dry)	5 568	2.7	35.	
Walnut	6.063 to 7,227	3. to 3.6	42.	
Willow	2.898 to 6,128	1.4 to 3.06	24.	

Resistance of Metals to Crushing.

Brass, yellow	10,304	5.15		Rennie.
CAST IRON.				
Buffery, No. 1, cold blast	93.366	46.68		E. Hodgkinson. Experiments made on prisms, base one inch square, hight 1½ inches.
Do. hot "	86 397	43.19		
Carron, No. 2, cold "	106.375	53 18		
Do. hot "	108.540	54 27		
Coed Talon, No. 2, c. b.	81.770	40.88		
Do. h. b.	82.734	41.36		
Carron, No. 3, cold blast	115.442	57.72		
Do. hot "	133.440	66.72		
Devon, No. 3, hot "	145.435	72.71		
Stirlings toughened iron 2d and 3d quality	122.304 to 144,256	61.15 to 72.12		
WROUGHT IRON, From	36.000	18.		Rankine's Mechanics.
WROUGHT IRON, To	40,000	20.		

RESISTANCE TO NATURAL AND ARTIFICAL STONES TO CRUSHING.

Description.	Crushing weight per square inch. In lbs.	Crushing weight per square inch. In tons of 2.000 lbs.	Weight in lbs of a cubic foot.	Authority.
Brick (common)..	{ 500 800	0.25 0.40	{ 130 135	Rennie.
Do. Fire.	1,700	0 85		Rankine's Mech.
Brickwork.....	{ 417 612	0.208 0.306	112	Edwin Clark.
Chalk	334	0.16	{ 116 174	Rennie.
Granite	{ 5 500 11,000	2.75 5 50	{ 164 166	Rankine's Mech.
Magnisian Lime. limestones stones. { Barnack.....	1 120	0.56	126	Experiments made on stones for the new houses of Parliament, England. Note—These weights are those which produced fracture. The absolute crushing weights were occasionally double those here given. (Mahan's Civil Engineering, 6th Ed.)
Chilmark	2,912	1 45	150	
Hamhill (silicious)...	1 545	0 77	140	
Oolites. { Bolsover	4,928	2 46	144	
Huddlestone	2,306	1.15	132	
Roach Abbey.....	1,680	0.84	131	
Park Nook	716	0.35	131	
Sandstones. { Ancaster.....	1,680	0 84	137	
Bath Box	1,254	0.62	113	
Portland.....	2 128	1.06	132	
Ketton.	1,545	0.77	126	
Craigleith.....	4,233	2.11	139	
Darley Dale.....	6,160	3.08	162	
Heddon.....	1,836	0.91	139	
Kenton	3,382	1.69	140	
Mansfield	1 971	0.99	143	
American sandstones { Belleville. N. J.	3,522	1 76	145	Experiments by R. J. Hatfield, Arch. N. Y. (Civil Engineer and Architect's Journal, May 1858.
Connecticut	3 319	1.65	150	
Dorchester	3 059	1.52	148	
Little Falls.....	2,991	1.49	145	
Caen Stone	1,088	0.54	138	
White Marble.....	5,000	2.5	168	
Rubble Masonry	4-10th of that of cut stone.			Rankine's Mech.

Results of Experiments made with Actual Weight on Materials used in the Brittania Bridge, January, 1848. (*E Clark.*)

BRICKWORK.

No. 1.—9 inch cube of cemented brickwork [Nowell & Co.] No. 1 (or best quality), weighing 54 lbs., set between deal boards; crushed with 44,654 lbs.=551 lbs. per square inch.

No. 2.—9 inch brickwork, No 1, weighing 53 lbs. set in cement; crushed with 49,633 lbs.=612 lbs. per square inch.

No. 3.—9 inch brickwork, No. 3, weighing 52 lbs, set in cement; crushed with 36,800 lbs.=454 lbs. per square inch.

No. 4.—9¼ inch brickwork, No. 4, weighing 55½ lbs., set in cement; crushed with 48,653 lbs =568 lbs. per square inch.

No 5.—9 inch brickwork, No. 4, weighing 54½ lbs., set between boards; crushed with 33,836 lbs.=417 lbs. per square inch.

Mean crushing weight per square inch=521 lbs.

"The last three cubes of common brick continued to support the weight, although cracked in all directions; they fell to pieces when the load was removed. All the brickwork began to show irregular cracks a considerable time before it gave way.

"The average weights supported by these bricks was 37½ tons (of 2,000 lbs.) per square foot, equal to a column 583 feet high of such brickwork."

SANDSTONE.

The average weight required to crush this material is 150 tons (of 2,000 lbs.) per square foot, equal to a column 2,351 feet high of this kind of stone.

LIMESTONE.

The crushing weight of this material is 527 tons (of 2,000 lbs.) per square foot, equal to a column 6,433 feet high of such stone.

In the London Builder for 1848, p. 177, there are given the results of two experiments on piers of brickwork, 9 inches square on the plan and 2 feet 3 inches high, formed of good sound "Cowley stocks," set in cement mortar of good quality. One pier was built with bricks laid

flat, the other with bricks on edge ; they were proved two days after with the following results :—The pier having the bricks laid flatwise compressed one-fourth of an inch, then cracked under a weight of 56,000 lbs., and broke to pieces with 67,200 lbs. The second one did not compress, cracked with a weight of 67,200 lbs., and broke to pieces with 78,400 lbs.

Prof. Mahan says that the permanent strain on stone should not exceed one-tenth of the crushing weights shown by experiments made on small cubes, measuring about 2 inches on a side. He also furnishes that part of the following table which exhibits the ratio of permanent and crushing strains on some noted European structures. We have appended some additional data obtained from the office of the U. S. Lighthouse Board.

	Name.	Permanent Strain in lbs. per square foot.	Crushing weight in lbs. per square foot.	Ratios.
	Pillars of the dome of St. Peter's, (Rome)	33 330	536,000	1 to 16
	" " " St. Pauls', (London)	39.450	537,000	1 to 13.6
	" " " St. Geneviève, (Paris)	60,000	456,000	1 to 7.6
	" " " Church of Toussant, (Angers)	90 000	900,000	1 to 10
	Lower courses of the Bridge of Neuilly	3,600	570,000	1 to 15.8
The lower courses of	U. S. Light House, "1st Order;" brick tower, 160 feet high	8,300	71,000	1 to 8.5
	U. S. Light House, "2d Order;" brick tower, 130 feet high	8.000	71,000	1 to 8.8
	Minot's ledge Light House, Boston Harbor, Granite tower	7,600	912,000	1 to 120
	Merchant's Shot tower, (Brick,) Baltimore; 246 feet high, 39 feet 10 inches base	14,600	71,000	1 to 4 8

STRENGTH OF PILLARS.

Very complete experiments under this head have been made by E. Hodgkinson, of Manchester, England. The following are some of the more important facts which are recorded :—

"1st. In all long pillars of the same dimensions, the resistance to crushing by flexure is about three times greater when the ends are flat than when they are rounded.

"2d. The strength of a pillar with one end rounded and the other flat (*Fig.* 2, see frontispiece) is the arithmetical mean between that of a pillar of the same dimensions with both ends rounded, and one with both ends flat. Thus, of three cylindrical pillars, all of the same length and diameter, the first having both its ends rounded (*Fig.* 1), the second with one endrounded and one flat (*Fig* 2), and the third with both ends flat (*Fig.* 3), the strengths are as 1, 2, 3, nearly.

"3. A long, uniform cast iron pillar, with its ends firmly fixed, whether by means of discs or otherwise, has the same power to resist breaking as a pillar of the same diameter and half the length, with the ends rounded or turned, so that the force would pass through the axis.

"4th. The strength of a cast iron pillar, made like an English 'connecting rod' (cruciform section +), was found to be very small, perhaps not half as strong as if cast in the form of a uniform hollow cylinder.

"5th. A pillar irregularly fixed, so that the pressure would be in the direction of the diagonal, is reduced to one-third of its strength."

The following are Mr. Hodgkinson's formula for the ultimate strength of pillars, in which

D—The external diameter, or side of the square, of the column in inches.

d—The internal diameter of hollow column in inches.

L—The length of column in *feet*.

W—The breaking weight in tons of 2,000 lbs.

Nature of Column.	Both ends being rounded, the length of the column exceeding fifteen times its diameter.	Both ends being flat, the length of the column exceeding thirty times its diameter.
Solid cylindrical column of Cast Iron	$W=16.6\frac{D^{3.76}}{L^{1.7}}$	$W=49.4\frac{D^{3.55}}{L^{1.7}}$
Hollow cylindrical column of Cast Iron	$W=14.5\frac{D^{3.76}-d^{3.76}}{L^{1.7}}$	$W=49.6\frac{D^{3.55}-d^{3.55}}{L^{1.7}}$
Solid cylindrical column of Wrought Iron	$W=47.9\frac{D^{3.76}}{L^{2}}$	$W=149.7\frac{D^{3.55}}{L^{2}}$
Solid square Pillar of Dantzic oak (dry)		$W=12.2\frac{D^{4}}{L^{2}}$
Solid square Pillar of red deal (dry)		$W=8.7\frac{D^{4}}{L}$

"The above formulas are correct only when the length is from 30 to 90 times the diameter, when the colums yield wholly by bending."

"When the columns are shorter than given in the tables, or when the material yields by crushing and bending, the following formula was found to represent the strength with sufficient accuracy. Let b=the breaking weight as computed by the formula for long columns; let c=the crushing weight of the material. Then will $W=\frac{bc}{b+\frac{3}{4}c}$."

"The strength of similar pillars is nearly as the area of the cross section."

Relative Strength of Long Columns of Cast and Wrought Iron, Steel and Timber, of the same Dimensions, representing the strength of Cast Iron by 1,000.

Cast iron.	1,000.
Wrought iron,	1,745.
Cast steel,	2,518.
Dantzic oak,	108.8
Red deal,	78.5

"Wet timber was found not to be one-half as strong as when dry."

When wrought iron tubes or cells, formed of plates connected together with angle iron, are used as struts or pillars, Messrs. Fairbairn and Hodgkinson state that for a single cell, when the thickness of the plates is not less than one-thirtieth ($\frac{1}{30}$) of the diameter, the ultimate resistance to a compressive strain is 27,000 lbs. per square inch of section; but when a number of cells exist side by side in one girder, it may be taken at from 33,000 to 36,000 lbs. per square inch.

The formulas given below are from Professor RANKINE'S *Applied Mechanics*, 1858; *they apply to Pillars of any length or any form of cross section.*

Let W=The weight or pressure on the pillar in lbs.
" A=The cross sectional area in square inches.
" L=The total length of pillar in inches.
" h=The diameter if round, or the least dimension across the section, if otherwise shaped.
" f=A co-efficient of the material in respect to compression
" a =do. do. do of flexure.

Then for pillars having flat ends, $$W=\frac{f\,A}{1+4a\left(\frac{L}{h}\right)^2}$$ ----No. 4.

For pillars with rounded ends, $$W=\frac{f\,A}{1+4a\left(\frac{L}{h}\right)^2}$$ .....No. 5.

The value of a for wrought iron=$\frac{1}{5,000}$
And for cast iron $a=\frac{1}{00}$

The values of (f) for different kinds of loads are as follows:

	Breaking Load.	Proof Load.	Working Load.
Wrought Iron	36,000	18,000	6,000 to 9,000
Cast Iron	80,000	26,700	13,300 " 20,000

Tables exhibiting the comparative strengths of wrought and Cast Iron Pillars of various lengths.

PILLARS WITH FLAT ENDS.

Breaking load in lbs. per square inch of the cross section.

$\frac{L}{h}$ =	10	20	26.4	30	40	50	60	70	80
Wrought Iron.....	34840	31765	29230	27700	23480	19670	16300	13800	11600
Cast Iron.....	64000	40000	29230	24620	16000	11000	8000	6060	4700

The above table was computed from formula No. 4.

PILLARS WITH ROUNDED ENDS.

Breaking load in lbs. per square inch of cross section.

(Computed from formula No. 5.)

$\frac{L}{h}$ =	10	13.15	20	30	40	50	60	70	80
Wrought Iron.....	31850	29200	23520	16360	11500	8370	6200	4800	3780
Cast Iron.....	40000	29200	16000	8000	4706	3000	2100	1600	1200

The strength of connecting rods of steam engines, and other similar appliances, where the ends of the rods are movable, may be estimated from the latter table.

Example.—What is the breaking weight of a wrought iron connecting rod 16 feet 8 inches long, having a diameter in the centre of 5 inches? Here $L = 200''$, and $h = 5''$ then $\frac{L}{h} = \frac{200}{5} = 40$, and looking in the table for the corresponding number, we find the breaking weight per square inch to be 11500 lbs. Now the area of a rod 5 inches in diameter, is about 19.6 square inches, hence the entire breaking weight is $11500 \times 19.6 = 225{,}400$ lbs. If we assume the safe weight to be $\frac{1}{4}$ of the breaking weight, we have $\frac{225400}{4} = 56350$ lbs. at the load which may be borne with safety.

COLLAPSE OF TUBES.

The practical rule at present much used is, where tubes are exposed to external pressure, to double that thickness which would be necessary for internal pressure.

Experiments made by Wm. Fairbairn, for the British Association, 1857, *on the Resistance of Tubes* (*iron*) *to Collapse.*

So far as these experiments have been conducted they indicate, 1st, that the strength of tubes of equal diameter and thickness varies inversely as their length; that is to say, if a tube one foot long sustains a given pressure, the same tube, if two feet long, would bear but one-half that pressure.

2d. Tubes of the same length and thickness, but of different diameters, the strength is inversely as the diameter.

This latter rule agrees with the ordinary and accepted theory on this subject.

The law for the thickness seems not to have been determined as yet.

In all the tubes experimented upon, the ends were secured in a manner somewhat analogous to the modes adopted in steam boilers, and it seems reasonable to conclude that this fact has much to do with the remarkable results obtained.

Table of Experiments on the Collapse of Tubes.

(The tubes collapsed with a loud report and a hissing noise.)

	No. of Experiment.	Diameter of the tube in inches.	Length of Tube.	Thickness of Tubes in inches.	Pressure of colapse in lbs. per sq. in.	
The discrepancies in experiments from No. 1 to No. 6, are due mainly to the modes of securing the tubes while being tried.	1	6	2' 6"	.043	48	6" tubes.
	2	6	2' 5"	.043	47	
	3	6	4' 11"	.043	32	
	4	6	2' 6"	.043	52	
	5	6	2' 6"	.043	65	
	6	6	2' 6"	.043	85	
	7	4	1' 7"	.043	170	4" tubes.
	8	4	1" 7"	.043	137	
	9	4	3' 4"	.043	65	
	10	4	3' 2"	.043	65	
The tube in experiment No. 12 may be considered as three distinct tubes, as it had two rigid rings soldered to the outside, the result of this simple contrivance was to increase the strength three-fold.	11	4	5' 0"	.043	43	
	12	4	5' 0"	.043	140	
	13	8	2' 6"	.043	39	8" 10" tubes. tubes
	14	8	3' 3"	.043	32	
	15	8	3' 4"	.043	31	
	16	10	4' 2"	.043	19	
	17	10	2' 6"	.043	33	
	18	12.2	4' 10½"	.043	11	
	19	12	5' 0"	.043	12.5	
	20	12	2' 6"	.043	22	

From experiments 21 and 22, on elliptical tubes, it will be seen how great is the reduction of strength when we deviate from a circular form, by comparing experiment 22 with experiment 23, it appears that the cylinder in this case has more than three times the strength of the elliptical tube, under otherwise similar conditions.

	Number of Experiment.	Diameters of tube in inches.	Length of tube.	Thickness of Tubes in inches.	Pressure of collapse in lbs. per sq. in.
Elliptical tubes.	21	14"x10¼"	5' 0"	.043	6.5
	22	20¾"x15½"	5' 1"	.25	127.5
	23	18¾	5' 1"	.25	420.
Tubes with lap and butt joint.....	24	9	3' 1"	.25	——
	25	9	3' 1"	.14	262.
	26	9	3' 1"	.14	378.

Resistence of Tubes to internal Pressure.

The tube tried in experiment 24 was not collapsed, owing to the deficiency of strength in the apparatus used, (a cast iron cylinder.) Mr. Fairbairn states that it was not capable of sustaining with safety over 500 lbs. per square inch.

Number of Experiments.	Diameters of tube in inches.	Length of tube.	Thickness of tubes in inches.	Bursting pressure in lbs. per sq. in.
27	6	4' 0"	.043	375
28	6	2' 6"	.043	230
29	6	2' 0"	.043	235
30	6	1' 0"	.043	475
31	12.13	5' 0"	.043	110

Punching and Shearing Wrought Iron.

The following experiments are recorded in the *London Artizan*, for 1858: They were made with an hydraulic press, which was constructed for cutting up scrap iron by direct pressure.

Table of experiments on Punching.

No. of experiment.	Diameter of Punch in inches.	Sectional area cut.		Pressure on Punch in tons of 2,000 lbs.		Remarks.
		Thickness and Circumference	Area in sq in's.	Total.	per sq. inch of area cut.	
1	1	0.51 × 3.14	1.60	40.	25.	25.2 mean.
2	1	0.98 × 3.14	3.08	77.6	25.3	
3	2	0.52 × 6 28	3.27	66.9	20.4	21.7 mean.
4	2	0.57 × 6.28	3.58	78.9	22.	
5	2	1.06 × 6.28	6.66	148.7	22.2	
6	2	1.52 × 6 28	9.55	209.	21.8	

Table of experiments on Shearing.

No. of experiment.	Direction of Shearing.	Sectional area cut.		Pressure on cutters in tons of 2000 lbs.		Remarks.	
		Thickness and breadth in inhs.	Area in square inches.	Total.	Per square inch of area cut.		
7	Flat	0.50 × 3.00	1.50	37.4	24.9	25 4 mean.	hamm'r'd scrap iron
8	Edge	0.50 × 3.00	1.50	38.7	25.8		
9	Flat	1.00 × 3.00	3.00	77.6	25.8	24.0 mean.	Parallel cutters for shearing.
10	Edge	1.00 × 3.00	3.00	76.3	25.4		
11	Flat	1.00 × 3.02	3 02	66.9	22.1		
12	Edge	1.00 × 3.02	3.02	69.5	23.0		
13	Edge	1 80 × 5.	10.2	235.8		Flanged tyre.	
14	Flat	0.56 × 3.00	1.68	23.7	14.1	Hammered Scrap Iron.	Inclined cutters for shearing. (Angle, 1 in 8.)
15	Edge	0.56 × 3.00	1 68	37.1	22.0		
16	Flat	0.90 × 3 37	3 03	30.7	10.0		
17	Edge	0.87 × 3.32	2 89	64.2	22.1		
18	Flat	1.06 × 3.02	3.20	56.6	17.5		
19	Edge	1.06 × 3.02	3.20	75.6	23.6		
20	Flat	1.52 × 3.03	4.61	93.7	20.3		
21	Edge	1.53 × 3 03	4 64	104.4	22 5		
22	Flat	1.39 × 4.50	6.25	100 4	16.0		
23	Edge	1.38 × 4.50	6.21	124.5	20.0		
24	Flat	1.73 × 5.30	9.17	171.5	18.7		
25	Edge	1 73 × 5.30	9.17	231.8	25.3		
26	Flat	1.56 × 6:00	9.36	156.7	16.8		
27	Edge	1.56 × 6.00	9.36	192 9	20.6		
28	Square	3.10 × 3.10	9.61	184.9	19.2	Hammered Iron.	
29	Square	3.10 × 3.10	9.61	174.1	18.1	Rolled Iron.	
30	Flat	1.80 × 5.00	10.20	111.1	10.8	Flanged tyre.	
31	Edge	1.80 × 5.00	10.20	207.7	20.3	Do. do.	
32	Edge	1.70 × 5.25	10.57	201.0	19.	Do. do.	

The table of experiments given below were made by means of weights applied to a pair of levers, giving a total leverage of 60 to 1. The weights were added gradually by a few lbs. at a time, until the hole was punched.

Experiments on Punching Plate Iron, [*Rolled*]

Diameter of hole in inches.	Thickness of plate in inches.	Sectional area cut through in square inches.	Total pressure on punch in tons of 2,000 lbs.	Pressure per square inch of area cut in tons of 2,000 lbs.
0.250	0.437	0.344	9 2	27.3
0.500	0.625	0.982	29.7	30.4
0.750	0.625	1.472	38.8	26.4
0.875	0.875	2.405	62.1	25.8
1.000	1.000	3.142	86.3	27.5

"The resistance of wrought iron to shearing may be estimated at from 70 to 80 per cent. of the resistence to direct tensile strain."

TRANSVERSE STRENGH OF MATERIALS.

Various theories have, from time to time, been proposed for the transverse strength of materials, but as yet scarcely any one has been entirely satisfactory or in strict accordance with the results shown by experiments. The ordinary and least objectionable one is that which assigns a neutral axis at the centre of gravity of the cross section of the beam, and supposes the material compressed on the concave side of beam and extended on the convex side when deflected by weight; the resistance to compression or extension of any element being directly as its distance from the neutral axis.

This theory has been variously modified by Galileo, Lebnitz, Hooke, Dr. Young, and others More recently Prof. W. H Barlow has shown that in a beam subjected to transverse strains, in addition to the resistances of extension or compression, there exists another element which he terms resistance of flexure.

Numerous experiments have been made by eminent engineers giving results which are generally, in ordinary construction, sufficient to guide our practice. Some fundamental theorems have been abundantly confirmed—such, for instance, as that in beams of equal length and breadth, the strength is as the square of the depth; and in beams of equal length and depth, directly as the breadth; so that the strength of a beam may be represented by $\frac{\text{breadth} \times \text{square of depth}}{\text{length}}$ multiplied by a co-efficient, varying according to the nature of the material.

For purposes of comparison, the latest authorities introduce into the formula of strength the total area of cross section. Thus, instead of writing $\frac{\text{breadth} \times \text{square of depth}}{\text{length}} \times$ co-efficient, they write $\frac{\text{area} \times \text{depth}}{\text{length}} \times$ co-efficient, and then

the comparative strength of beams having various proportions of cross section, becomes manifest by a simple inspection of the co-efficients, which are determined by experiment.

Messrs. Hodgkinson and Fairbairn, in giving the results of their experiments on iron beams, have only introduced the area of the bottom flange as an element of calculation. Mr. Edwin Clark, in his work on the Britannia and Conway bridges, adopts the total area of cross section as a basis of calculation; also, S. Hughes, C.E., who has written a series of valuable papers on the strength of beams for the London Artizan Journal, 1857 and '58.

In applying the different formulas for the strength of beams, which are given in this article—

Let L=The length of the beam in feet, clear of supports.
Let A=The total area of the cross section in square inches.
Let d=The depth of beam in inches.
Let b=The breadth of beam in inches.
Let k=The co-efficient of breaking weight.
Let S=The co-efficient of safe weight.

W will be the breaking weight in lbs. or tons, according as k is taken in lbs. or tons. If S be substituted for k, the result, W will be the safe weight.

Figures 4, 5, 6 and 7, on the lithograph sheet, illustrate how the strength of a beam is affected by placing the weights and supports differently. The number of balls representing the amount of weight; those shown as shaded represent the amount of weight which can be borne when it is concentrated in one place, and those shown with dotted lines, the amount which can be borne when the weight is uniformly distributed.

The co-efficient k, *as given in the tables, is the breaking weight of a beam* 1 *foot long*, 1 *inch deep, and* 1 *square inch area of cross section* (*fixed at one end and loaded at the other.*)

The breaking weights of beams variously circumstanced are as follows:—

Fixed at one end, and loaded at the other,	$W=\frac{kbd^2}{L}$ or $W=\frac{kAd}{L}$	No. 6
Fixed at one end, and uniformly loaded,	$W=2\left(\frac{kbd^2}{L}\right)$ or $W=2\left(\frac{kAd}{L}\right)$	No. 7
Supported at both ends, and loaded at the centre,	$W=4\left(\frac{kbd^2}{L}\right)$ or $W=4\left(\frac{kAd}{L}\right)$	No. 8
Supported at both ends, and uniformly loaded,	$W=8\left(\frac{kbd^2}{L}\right)$ or $W=8\left(\frac{kAd}{L}\right)$	No. 9
Fixed at both ends, and loaded in the centre, as in Fig. 7,	$W=6\left(\frac{kbd^2}{L}\right)$ or $W=6\left(\frac{kAd}{L}\right)$	No. 10
Fixed at both ends, and uniformly loaded, as in Fig. 7,	$W=12\left(\frac{kbd^2}{L}\right)$ or $W=12\left(\frac{kAd}{L}\right)$	No. 11

Beams of uniform strength.

In order that the strength may be uniform throughout a beam, it is obviously necessary that the cross section at different places should be proportioned to the amount of strain upon it. This strain is always directly as the weight multiplied by the length of the arm or lever with which it acts, and is what is usually termed by writers on mechanics, the "moment of rupture." Hence, when we know the moment of rupture for any point on a beam, we can at once determine the proper dimensions at that place; presuming the co-efficient of the material known.

Case 1st.—Beam fixed at one end and loaded at the other. Here, the moment of rupture is $= W \bowtie L$, and the strength

of the beam [which should of course be equal to the strain upon it,]$=kbd^2=WL$ - - - Equation, No. 12

To ascertain how the breadth must vary when the other parts remain constant, we find the value of $b=\frac{W}{kd^2} \times L$ —No. 13.

as b is to be found for various lengths, while $\frac{W}{kd^2}$ remains constant, it will be seen that this is an equation to a straight line, the breadth being directly as the length. *See Fig.* 9.

When the breadth is constant, we find the value of

$$d=\sqrt{\frac{W}{kb} \times L}=\sqrt{\frac{W}{kb}} \times \sqrt{L}. \quad - \quad - \quad - \quad \text{No. 14}$$

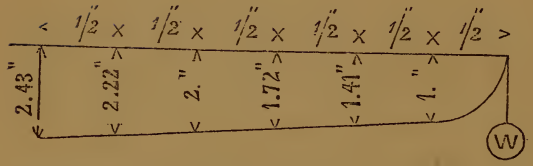
That is, the depth varies as the square root of the length, and No. 14 is an equation to the common parabola, with the vertex at the free end of beam, [See Fig 8,] the parameter being $\frac{W}{kb}$. If we wish to determine the ordinates d, for different lengths or distances from the weight, we extract the square root of $\frac{W}{kb}$ as already indicated, and as the quantity thus obtained remains constant, we multiply it into the different square roots of L, and the results are the corresponding ordinates.

Example —A cast iron beam fixed at one end and loaded at the other is to be 3 ft. long, and have a uniform breadth of 1 inch. The breaking weight to be 1,100 lbs. What must the values of d, at the distances $\frac{1}{2}$ ft., 1 ft., $1\frac{1}{2}$ ft, 2 ft., $2\frac{1}{2}$ ft. and 3 ft. from the weight?

Here $W=1{,}100$, $b=1$ and k for average, cast iron may be taken equal to 550.

Hence $\sqrt{\frac{W}{kb}}=\sqrt{\frac{1100}{550 \times 1}}\sqrt{2}=1.41.$

Now when L=3' ft. $d=1.41\sqrt{3}=2.43''$
" " " $2\frac{1}{2}$ ft. $d=1.41\sqrt{2.5}=2.22''$
" " " 2 ft. $d=1\ 41\sqrt{2}=2''$
" " " $1\frac{1}{2}$ ft. $d=1.41\sqrt{1.5}=1.72''$
" " " 1 ft. $d=1.41\sqrt{1}=1\ 41''$
" " " $\frac{1}{2}$ ft. $d=1.41\sqrt{5}=1.$



When the cross sections of the beam are all similar, as when they are circles, squares, elipsis, etc., or when the breadth bears a constant ratio to the depth, then, expressing this ratio by n, we may put $b=nd$ and substituting this value in equation No. 12 we find $WL=k(nd)d^2=knd^3$.

Whence $d=\sqrt[3]{\frac{WL}{kn}}=\sqrt[3]{\frac{W}{kn}}\bowtie\sqrt[3]{L}$ - No. 15.

That is, d varies as the cube root of the length and as the breadth bears a constant ratio to the depth, the curves on all sides are similar and are cubic parabolas See Fig. 10.

Case 2d.—Beam fixed at one end and *uniformly loaded.*

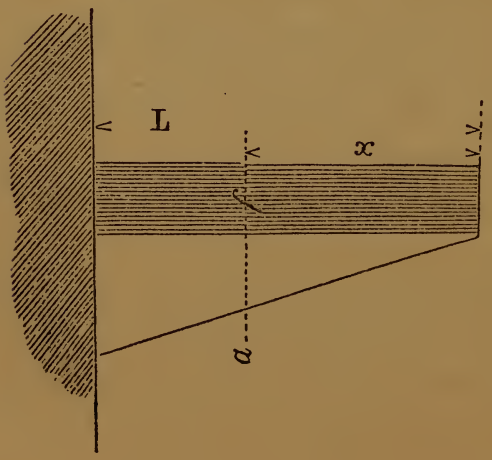


Fig. E.

By inspection of Fig. E, it will be seen that any section, as a—$\int$ will only be strained by the weights which lie beyond it. If we represent by q the weight per unit of length, then the total weight for any distance x from the free end of beam will be qx, and as this weight acts through its centre of gravity at a distance $\frac{x}{2}$; *The moment of rupture will be

$$qx \bowtie \frac{x}{2} = \frac{qx^2}{2} = kbd^2 \quad \text{- - - - - No. 16.}$$

When the breadth is constant,

$$d = \sqrt{\frac{q}{k}x^2} = \sqrt{\frac{q}{2kb}} \bowtie x \quad \text{- - - - No. 17.}$$

That is, d varies directly as the length or distance from the free end of beam, and consequently the line obtained by different values of d, is straight. See Fig. 11.

When the depth is constant.

$$b = \frac{q}{2kd^2} \bowtie x^2 \quad \text{- - - - - No. 18.}$$

Here b varies as the square of the length or distance from free end of beam, and the curve thereby produced is the common parabola with the vertex at the free end of beam. [When the ordinates are laid off equally on each side of the centre line, the nature of the curve is not changed, but a pair of parabolas are obtained, as in fig. 12]

When the cross sections are similar figures or when $b = nd$, then $knd^3 = \frac{q}{2}x^2$ - - - - No. 19.

whence $d = \sqrt[3]{\frac{q}{2kn}x^2} = x^{\frac{2}{3}} \times \sqrt[3]{\frac{q}{2kn}}$ - - No. 20.

Here d varies as the cube root of the square of the length $\bowtie$, and the lines of the top, bottom and sides of

---

* This may also be obtained by direct integration, $q\int_0^x x dx = \frac{qx^2}{2}$ $qxdx$ being the moment of a very small portion of the length.

beam are semi-cubical parabolas, with the vertices at the free end of beam. (*See Fig.* 13.)

Case 3*d*.—Beam supported at both ends, and loaded in the centre, or at any intermediate point. When the weight is on the middle of beam, each support receives one-half of it, and the moment of rupture—

$$= \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4} = kbd^2 \quad \text{No. 21}$$

When the weight is at any intermediate point, say at a distance x, from one support (*see Fig.* 14), then the pressure $P_1 = \left(\frac{L-x}{L}\right) W$. and $P_2 = \frac{x}{L} W$.

The moment of rupture $= P_1 x = P_2 (L-x) = kbd^2$; also

$$P_1 x = x \left(\frac{L-x}{L}\right) W = \frac{Lx - x^2}{L} W = kbd^2, \quad \text{No. 22.}$$

And the strength of the beam under these circumstances varies inversely as the product of the two segments in which its length is divided, viz., x and $L-x$.

By inspecting Figs. 5 and 6, it will be readily seen that Case 3d is but a variation of Case 1st, the same shapes being necessary for beams similarly circumstanced. For instance, when the depth is constant the sides are straight lines, the plan of the beam being two triangles with their bases abutting each other at the point of application of the weight. (*See Fig.* 15.)

When the breadth is constant the depth varies as the ordinates of the common parabola, (*see Fig.* 14), and when the cross sections are similar figures, the curved sides are cubic parabolas.

Case 4*th*.—Beam supported at both ends, and uniformly loaded.

(*See Fig* 18 for illustration of this case.)

The moment of rupture for any section at a distance x from a support, may be determined as follows:—

Let q = weight on a unit of length, the $qL = W$, and the pressure at each support $= \frac{W}{2} = \frac{qL}{2}$. It will be seen by inspecting the figure that $\frac{W}{2}$ (or the resistance of support), acting at the end of the arm x, tends to break the beam in one direction, while the weight which lies between the support and a—f, and which is equal to qx, acts at its centre of gravity on an arm $\frac{x}{2}$, and tends to break the beam in the opposite direction; hence the actual moment of rupture must be the difference between these two moments; that is,

$$kbd^2 = \left(\frac{qL}{2} \times x\right) - \left(qx \times \frac{x}{2}\right) = \frac{q}{2}\left(Lx - x^2\right), \quad \text{No. 23.}$$

When the depth is constant, $b = \frac{q}{2kd^2}\left(Lx - x^2\right)$

*This is an equation to the common parabola, the origin of the co-ordinates being at one of the supports; and the vertex (or vertices of a pair, when the ordinates are laid off equally on each side of the centre line of beam) at the middle of the length. (*See Fig.* 17)

When the breadth is constant, $d = \sqrt{\frac{q}{2kb}} \times \sqrt{Lx - x^2}$

†This is an equation to an ellipse; the origin of the co-ordinates being at one of the supports. (*See Fig.* 16.)

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* *Vide* Loomis' Analytical Geometry, Art. 128.

† " " " " Art 70.

When all the sections are similar figures, $knd^3 = \frac{q}{2}(Lx - x^2)$

No. 24.

$$d = \sqrt[3]{\frac{q}{2kn}} \times \sqrt[3]{Lx - x^2}$$

Provision to be made at the Ends of the Beams for the direct Shearing Force of the Weight.

The force tending to shear off the beams at the ends is directly as the weight or load, and if we neglected the weight of the beam itself, it remains the same for any length of beam.

Representing by f the shearing force of one square inch of the material, then for beams fixed at one end and loaded at the other, the area of cross section at the free end should be at least equal to $\frac{W}{f}$.

When the beam is sustained at both ends, the area should be $\frac{W}{2f}$.

It is also generally necessary in practice to modify the ends of the beam, so as to obtain suitable bearing surface, &c.

Table of CO-EFFICIENTS *of the transverse strength of materials having rectangular sections.*

Material.	Values of K in lbs. avoirdupois.	Values of K in tons of 2,000 lbs.	Values of S in lbs. or co-efficients for safe weight $S=\frac{k}{4}$	Authority.
TIMBER.				
Ash, English	168.	.084	42.	Papers of the Royal Engineers, Vol. V.
" American	149.	.074	37.	
" " Swamp	97.	.048	24.	
" " Black	71.	.035	17.	
Beech, English	129.	.064	32.	
" American white	115.	.057	28.	
" " red	144.	.072	36.	
Birch, common	160.	.080	40.	
" American black	171.	.085	42.	
" " yellow	111.	.055	27.	
Cedar, Bermuda	120.	.060	30.	
" American white	63.	.031	15.	
Elm, English	65.	.032	16.	
" Canada rock	164.	.082	41.	
Hickory, American	177.	.088	44.	
" " bitter nut	122.	.061	30.	
Oak, English	141.	.070	35.	
" American white	145.	.072	36.	
" " red	140.	.070	35.	
" " live	155.	.077	38.	
" Adriatic	122.	.061	30.	
Pine, Virginia	121.	.060	30.	
" Memel	112.	.056	28.	
" American white	102.	.051	25.	
" " red	127.	.063	31.	
" " yellow	98.	.049	24.	
" " pitch	143.	.071	35.	
Spruce	112.	.056	28.	
" American	86.	.043	21.	
Norway spar	122.	.061	30.	
Deal, Christiana	130.	.065	32.	
Canada balsam	93.	.046	23.	
Hemlock	95.	.047	23.	
Larch	111.	.055	27.	
" Amer. or Tamarak	75.	.037	18.	
Lignum Vitae	167.	.083	41.	
Iron wood, Canada	150.	.075	37.	
Soft Maple, Canada	141.	.070	35.	
Teak	170.	.085	42.	Barlow.
Mahogany, Span. seasoned	106.	.053	26.	Tredgold.
" Honduras "	159.	.079	39.	
Chestnut tree	112.	.056	28.	Ebbels.
Walnut, green	121.	.060	30.	
Willow	91.	.045	22.	Tredgold.
New England pine	91.	.045	22.	Barlow.
Pitch pine	136.	.068	34.	

	Values of K in lbs.	Values of K in tons of 2 000 lbs	Values of S in lbs.	
AMERICAN STONE.				
Blue Stone flagging.....	31.	.015	7.7	From experiments made by R. G. Hatfield. Architect New York. Jour. Frank. Inst. 1858.
Quincy granite	26.	.013	6.5	
Little Falls freestone.....	24.	.012	6.	
Belleville, N.J. freestone....	20.	.010	5.	
Granite bl. (another quarry)	18.	.009	4.5	
Connecticut freestone.....	13.	.0065	3.2	
Dorchester do.	11.	.0055	2.5	
Aubigny do.	9.	.0045	2.2	
Caen do.	6.	.003	1.5	
CAST IRON.				
Ponkey, No. 3, cold blast....	654.	.327	163.	E. Hodgkinson, and W Fairbairn.
Devon, No. 3, hot blast.....	604.	.302	151.	
Carron, No. 3, " " . ..	592.	.296	148.	
Butterley,..	564.	.282	141.	
Buffery, No. 1, c. b.	520.	.260	130.	
Devon, No. 3, c. b.	504.	.252	126.	
Coedtalon, No. 2, h. b	468.	.234	117.	
Do. do. c. b.	462.	231	115.	
	520.	.260	130.	W. H. Barlow, 1858.
	610.	.305	152.	
	540.	.270	135.	
Common pig iron...	500.	.250	125.	Maj. Wade, U. S. A.
Good common castings.....	625.	.312	156.	
WROUGHT IRON.				
	798.	,399	199.	Drewry.
	714.	.357	178.	Lillie.
	714.	.357	178	Edwin Clark.
	577.	.288	144.	Barlow.
Swedish bar.	750.	.375	187.	General Millar.
Hammered iron bar..... ...	1869.	.934	467.	Mersey steel & iron works, Eng. 1857.
Hammered puddled steel bar	2709.	1 354	677.	

Relative Strength of Wrought and Cast Iron Beams.

Mr. Hughes, in his paper before referred to, remarks as to the transverse strength of wrought iron :—

"In comparing this with cast iron, it is important to observe the remarkable distinction which characterises the two separate forms of iron. Cast iron bends but little before it breaks, and its elasticity is injured by a very small deflection. Wrought iron, on the contrary, will bend considerably without breaking, and will bend much more than cast iron before its elasticity is destroyed.

"Owing to this distinction the breaking weight of cast iron is usually considered, and the safe weight assumed as

a certain fixed proportion of the breaking weight; but in wrought iron the breaking weight is seldom determined. It is sufficient to ascertain the weights which produce certain deflections, and particularly that which injures elasticity; the safe permanent or occasional load must be considerably less than the latter."

Taking the results of numerous experiments, and the practice of the best engineers in Europe and this country, as our guide—we may adopt as co-efficients of safe weights for beams having rectangular sections—

$$\left.\begin{array}{r}\text{For cast iron, } S=100\\ \text{And for wrought iron, } S=150\end{array}\right\} \text{ in lbs.}$$

Example.—A weight of 1,029 lbs. is to be supported at the end of a beam 7 feet long and 8 inches deep. How thick should it be to sustain the above weight safely, if made of cast iron; and how thick if made of wrought iron? For this description of beam, $WL=SAd=Sbd^2$, and

$$b=\frac{WL}{Sd^2}.$$

$W=1{,}029$ lbs.
$L=7$ feet.
$d=8$ feet.
For cast iron, $S=100$.
" wrought " $S=150$.

Thickness of cast iron beam—

$$=b=\frac{1{,}029\times 7}{100\times 64}=1\tfrac{1}{8}''$$

Thickness of wrought iron beam—

$$=b=\frac{1{,}029\times 7}{150\times 64}=\tfrac{3}{4}''$$

Experiments of Mr. Macperson, of Philadelphia, Penn., on the Transverse Strength of certain Alloys, 1855.

All the bars were cylindrical, and of the following dimensions :—Extreme length, 25 inches, and turned to a diameter of $2\frac{1}{16}$ inches.

The weights were applied in the centre ; the distance between the supports was 24 inches.

The formula for the breaking weight of beams, supported at both ends and loaded in the middle, is $W = \frac{4kAd}{L}$. (*See No.*

Hence, $k = \frac{WL}{4Ad} = \frac{2}{4 \times 3.34 \times 2.0625} W = .0764\,W$

$L = 2.$

$A = 3.34$ square inches.

$d = 2.0625''$

No.	Nature of Mixture.	Breaking weight.	Ultimate deflection in inches.	Values of k in lbs.	Values of $S = \frac{k}{4}$
1	10 lbs. Copper, 10 oz. Tin, 12 oz. Zinc, 8 oz. Lead	6874	7⅜"	525.	131.
2	6 lbs. Copper, 4 lbs. Zinc.....	6489	2.	495.	123.
3	2 " " 1 " "	6158	7 9-16	470.	117.
4	9 " " 1 " Tin	7802	4¼	596.	149.
5	10 " " 1 " " 4 oz. Zinc.....	10530	7½	804.	201.
6	Same as No. 5, but cast some weeks later, it endured without breaking...	11695	7½	893.	223.
7	8½ lbs. Copper, 8 oz. Tin, 1 lb. Zinc	6461	5⅜	493.	123.
8	Same as No. 1, with the addition of 1 lb. of Tin to every 12 lbs. of mixture	5333	¼	407.	101.
	A piece of refined bar Iron of the same length and diam deflected slowly to 4 inches and sustained 12375 lbs. and by increasing to 17000 it deflected to 5½.	17000	5½	1298.	324.

Experiments on Cast Iron Beams, having Cross Sections of Various Forms.

In the valuable experiments by Messrs. Hodgkinson and Fairbairn, several theories that had been sanctioned by high authority were shown to be erroneous. " It had been held by Tredgold and others, that a beam of the T section was equally strong whether the rib or the flange was uppermost

but Messrs. H. and F. found that in a beam of this shape, supported at both ends with the rib downwards, the breaking weight was 266 lbs., and when the rib was placed uppermost, the breaking weight was 1,050 lbs., or nearly 4 times as strong as before. It had also been held by earlier writers, that the elasticities of cast iron beams would remain perfect until loaded with nearly one-third of the breaking weight. But Mr. Hodgkinson found, by his experiments, that a visible set was produced in some cases with a weight not exceeding one-fiftieth to one-eightieth of the breaking weight, and concludes that there is no permanent weight, however small, that will not injure the elasticity of cast iron." He also ascertained from a large number of carefully conducted experiments, that the beam of greatest strength, with a given amount of material, was obtained by making the flange which is exposed to tensile strain six times the area of the flange subject to compression, the two flanges being connected with a thin vertical rib. This result has since been shown theoretically, reasoning from the now well-known fact, that cast iron offers about six times the resistance to compression that it does to extension.

The accompanying table contains some of the more important experiments of Messrs. Hodgkinson, Barlow and Fairbairn, and the co-efficients deduced from them exhibit the relative advantage of the various sections.

Prof. Barlow's experiments show that large sections are proportionately weaker than small ones.

It should not be forgotten that, in large beams, one-half of the weight of the beam itself should be added to the amount which is required to be borne. This becomes a very important item in designing girder bridges. Of course the *exact* weight cannot be known until the section is determined, but for all practical purposes it may be generally approximated.

Experiments on the

of cast iron beams of various sections,

Shape and position of cross section.	*L* Length or clear span in ft, & ins.	*d* Depth of beam in inches.	*A* Total area of cross section in sq. in's.	Ratio of bottom flange to top flange.	Ratio of bottom flange to the total area of cross section.
	5' 0"	1.01	1.032	——	——
"	" "	2.	4.01	——	——
	" "	1.44	1.04	——	——
"	" "	2.83	4.02	——	——
	" "	1.12	.989	——	——
"	" "	2.5	4 97	——	——
"	" "	2.2	3.78	——	——
Usually called the Tredgold beam.	4' 6"	5⅛	2.82	1 : 1	1 : 4
Sometimes called the Fairbairn beam.	" "	"	3.2	——	1 : 2.73
The Hodgkinson beam.	" "	"	2.87	2 : 1	1 : 3
"	" "	"	3.02	4 : 1	1 : 2.5
"	" "	"	3.37	4½ : 1	1 : 2.15
"	" "	"	4.5	4 : 1	1 : 2
"	" "	"	5.0	5½ : 1	1 : 1.73
"	" "	"	6.4	6 : 1	1 : 4.5
"	" "	"	6.5	6.73 : 1	1 : 1.51
"	" "	"	5.41	6 72 : 1	1 : 1.62
"	19' 0"	22.	39.6	2.75 : 1	1 : 3 5
"	22' 6"	17.	24 5	4.9 : 1	1 : 2
"	27. 3'	18.	29.1	4.6 : 1	1 : 2

Transverse strength

[supported at both ends and loaded at the centre]

W Breaking weight in lbs. avoirdupois.	k Values of k in lbs. in the formula $W=\frac{4kAd}{L}$	k Values of k in tons of 2,000 lbs. in the formula $W=\frac{4kAd}{L}$	S Values of S or coefficients of *safe* weight in lbs. $S=\frac{k}{4}$	Authority.
527.	658.	.329	164.	W. H. Barlow. See Civil Engineers and Architects' Journal, 1858.
3478.	542.	.271	135.	
449.	374.	.187	93.	
2988.	330.	.165	82.	
474.	538.	.269	134.	
4143.	417.	.208	104.	
3132.	471.	.235	117.	
6678.	520.	.260	130.	E. Hodgkinson See Artizan Journal, 1857-8. Hughes' Papers.
8270.	568.	.284	142.	
7368.	564.	.282	141.	
8270.	602.	.301	150.	W. Fairbairn. See Artizan Jour. '57-8
10727.	700.	.35	175.	
14462.	707.	.353	176	
16730.	736.	.368	184.	
26784.	896.	.448	224	
23249.	786.	.393	196.	
21009.	854.	.427	213.	
116480.	635.	.317	158.	
50064.	670.	.335	167.	
65632.	855.	.427	213.	

Example—A "Hodgkinson" beam, supported at both ends, has an area of cross section at the centre of 20 square inches; the bottom flange having 12 square inches, and the top flange 2 square inches, or the ratio to each other as 6 to 1.

The depth of the beam is 15 inches, and the distance between the supports 18 feet. What is the breaking weight at the centre in tons of 2,000 lbs?

For beams supported at both ends, and loaded at the centre, we have, $W = \frac{4kAd}{L}$

$A = 20$, $L = 18$, and $d = 15$.

Referring to the table, we find for a beam proportioned as above, $k = .448$.

Hence, $W = \frac{4 \times .448 \times 20 \times 15}{18} = 29.8$ tons of 2,000 lbs.

Experiments, supported at both ends and

Shape and posit	k values of k in lbs. in $W=\frac{4kAd}{L}$	k values of k in tons of 2000 lbs	S in lbs co efficient of safe w'gts $S=\frac{k}{4}$	Authority.
Rolled Bea	675.	.337	168.	W. Fairbairn.
" " "	674.	.337	168	
" " "	808.	.404	202.	
3½" 16"	603.	.301	150.	

Experiments on the Transverse Strength of WROUGHT IRON *beams of various sections, supported at both ends and loaded at the centre.*

Shape and position of cross section	L Length or clear span.	d Depth of Beam in inches.	A Total area of cross section in sq. inches.	Area of top flange in square inches.	Area of bottom flange in sq. inches.	Greatest w'gt within limits of elasticity in lbs.	W in lbs wt. which destroys elasticity.	Weight in lbs producing deflections e	e Deflections in inches.	k values of k in lbs. in $W=\frac{8kAd}{L}$	k values of k in tons of 2000 lbs	S in lbs coefficient of safe w'gts $S=\frac{k}{4}$	Authority.
Rolled Beam	11' 0"	8.38	6.295	2.5	1.52		12955			675.	.337	168.	W. Fairbairn.
" " "	10' 0"	9 44	7.442	2.75	1 89		18962			674.	.337	168	
" " "	10' 0"	9.42	7.59	2.75	1.8		23046			808.	.404	202.	
16" 3/8" 3½"×3½"×½" / 3½"×3½"×½" — Fairbairn Plate beam.	24' 0"	16.	19.9	7.	7.		31980			603.	.301	150.	
4' 6" × 2' 8" — Model Tube for the Britannia Bridge.	75' 0"	54.	45.824	24 024	12 8		126112			955.	.477	238.	Edwin Clark.
8 15/16" — Rolled Beam	21' 0"	8 15/16	8.772	2.836	2.836	9908	20799	2559 4865 6882 9908 10916	.258 .505 .725 1.064 1.184	1391.	.695	347.	Phenix Iron Company Philadelphia, Penn. 1859.
12½" — 6"×⅝" / 3½"×2½"×⅜" / ⅜" / 3½"×3½"×½" / 12"×⅜"	28' 6"	12.5	25.7	9.7	11.5	24080		12616 24080	1. 2.				Thomas Davies, F. A. I. S. See Civil Engineer and Architects' Journal, 1857—page 20. (As all the beams which Mr. Davies experimented upon were intended for actual use the *ultimate* strengths were not ascertained.)
12⅛" — 2½"×2½"×⅜" / ⅜" / 1¾"×1¾"×5-16" / 4"×⅜"	28' 6"	12⅛				19040		11410 19040	1.125 2.25				
16½" — 7"×⅜" / ⅜" / 3"×3"×7-16" / 2"×2"×⅜" / 4½"×½"	22' 6"	16½	19.8	8 75	5.25	30240		18170 30240	7/16 7/8"				
Above beam inverted	" "	"	"	5 25	8 75	30240		19500 30240	7/16 13/16				
10" — 4½" / ½" / 3⅛" — Rolled Beam	10' 3"	10"	10	2.	4.	22904		22904	7/16				

Co-efficients for the Transverse Strength of Iron Beams, from Edwin Clark's work on the Conway and Britannia Bridges.

CAST IRON.

	Values of k in lbs.
Uniform rectangular tubes, - - -	511
" elliptical " - - - -	564
" circular " - - - -	533
" square " - - - -	637
Small rectangular bars, - - - -	634
Large " " - - - -	420
Small round bars, - - - - -	373

WROUGHT IRON.

Rectangular tubes, with thick top and bottom plates, and thin sides, as in the Britannia and Conway bridges, . - - - -	1,246
Rectangular welded tubes, uniform thickness, -	1,096
Elliptical " " " " . -	1,040
Circular " " " " -	975
Round rivetted tubes, - - - -	608
Oval " - - - -	714
Rectangular " - - - -	843
New rectangular bars (solid), - - -	714

Warren or Half Lattice Girders.

This variety of lattice girder seems justly to obtain at present the preference of engineers. Theoretically, it has been declared to be the most perfect beam, giving the maximum of strength with a minimum of material. Owing to practical considerations, however, the Fairbairn or wrought iron plate beam still continues a formidable rival. For a full investigation of the subject, see Prof. Rankine's

"Applied Mechanics," London, 1858, and the Civil Engineer and Architect's Journal for 1853 and '55.

We propose here to furnish only some of the more prominent facts.

There are four cases which present themselves—

1st. When a beam is fixed at one end, and loaded at the other.

2d. The same beam uniformly loaded.

3d. When a beam is supported at both ends, and loaded in the centre.

4th. Beam supported at both ends, and uniformly loaded.

Let L = The total length of unsupported part of beam, or the clear span.

Let d = Depth of beam.

Let n = Number of diagonals, as 1, 2, 3, 4, &c., (see Fig.)

Let Φ = Angle contained between the diagonal and a vertical line.

Let W = Total weight.

Let S = Horizontal strain (on either the upper or lower strips).

Let Σ = Strain on a diagonal.

Case 1st. Beam fixed at one end, and loaded at the other. (See Fig. 20.)

The horizontal strains are greatest at the point of support, and decrease directly as the distance from that point, or increase directly as the distance from the weight.

At any point distant n diagonals from the weight, $S = W.\ n.$ tang. Φ, but as $n.$ tang. $\Phi . d = L$, and n tang $\Phi = \frac{L}{d.}$

We may substitute this latter value in the equation first given, and we obtain—

$$S = \frac{WL}{d} \quad \text{No. 25}$$

It will thus be noted that the horizontal strains do not depend on any values of Φ; they are the same for any angle.

The strain on each diagonal remains constant throughout the beam, and $\Sigma = W.$ secant Φ. - - - No. 26

Case 2d. Beam fixed at one end, and *uniformly loaded.*

Representing by q, the weight per unit of length, and by x any given distance from the free end of beam. Then the horizontal strain at a distance x, will be $S = \frac{q.x^2}{2.d}$, that is, the horizontal strain increases towards the support as the square of the length; the other elements being constant. When x becomes equal to L, then $q\,x^2 = W.L$. Hence, at the point of support, $S = \frac{W.L}{2.d}$, - - - - - - No. 27

The strain on any diagonal increases directly as its distance from the free end of beam. We have, $\Sigma = q.x.$ secant Φ, - - - - - - No. 28

Case 3d. Beam supported at both ends, and loaded at the centre.

This is but a variation of case 1st, as was noticed in regard to solid beams.

The horizontal strain is greatest at the centre, and decreases toward the supports directly as the length.

$S = \frac{W.L}{4.d}$ = strain at the centre, - - - - No. 29

The strain on the diagonals is constant, and

$\Sigma = \frac{W}{2}$. secant Φ, - - - - - - No. 30

Case 4th. Beam supported at both ends, and *uniformly loaded.*

q being the weight per unit of length; the horizontal strain at any point at a distance x from one of the supports

is $S=\frac{q.(Lx-x^2)}{2.d}$, - - - - - - No. 31

When x becomes equal to one-half the length of beam, or $=\frac{L}{2}$, then $S=\frac{q}{2.d}\left(\frac{L^2}{2}-\frac{L^2}{4}\right)=\frac{q}{2.d}\times\frac{L^2}{4}$, but $qL=W$, hence

$S=\frac{WL}{8d}$ = horizontal strain at the centre of beam.

The strain on the diagonals is least at the centre, and increases toward the supports directly as the distance from the centre.

$\Sigma=\left(\frac{L}{2}-x\right)q.$ secant Φ, - - - - No 32

In illustration of this fact, see Fig. 21, where the strains are represented by arrows in magnitude and direction, and the weight by balls suspended from each joint.

The facts embodied in the foregoing formulas for Warren girders, were also ascertained experimentally by W. T. Doyne and W. B. Blood, civil engineers, who constructed models for the express purpose of discovering the agreement between theory and fact. (See Civil Engineer and Architect's Journal, 1853, p. 20)

Trussed Cast Iron Girder.

Mr. Fairbairn and others have investigated and experimented upon this form of girder. The results obtained show that there is little, if any, advantage over the simple beam, while the combination deceives by leading persons to make false estimates of its strength.

Owing to the different rates of extension of wrought and cast iron, it commonly happens that, in trussed girders, the casting breaks before the tension rods are fully strained. Again, if the tension rods are highly strained at first, they generally yield with weights which might be borne by the beam itself. Mr. Fairbairn found the most favorable conditions of strength to be when the tension rods have a moderate initial strain upon them; but the gain, even then, is too small to warrant the use of the tension rods at all. He found, also, that the cost of the trussed, compared with the simple girder of equal strength, was as one to two-thirds; that is, the trussed girder was about 50 per cent. more costly. [See Fairbairn's Treatise on wrought and cast iron, 1854.]

About ten or twelve years ago, in England, there were numerous failures of trussed cast iron girders; one of the most remarkable was that of the bridge across the river Dee, on the line of the Chester and Holyhead railway. It consisted of three spans, each 98 feet in the clear; it fell under a passing train in 1847. This and other accidents of a similar nature diminished the confidence of engineers in such combinations, and we are informed by English writers that there have been scarcely any bridges of this description built within the last ten years; the wrought iron plate and lattice girders are now rapidly superseding other varieties.

The account of the following interesting experiment, which was made at Mr. Cubitt's works on the Thames, we extract from the London Builder of 1848:

"The girder was of cast iron, 28 feet long, 10 inches deep, parallel, and of the same section throughout. A strong frame was cast upon the ends of it, in order that the inclination of the tension rods might be varied, or, rather,

that the ends of them might be raised to different heights above the bottom flange. [See Fig. 19.]

"The tension rods were of wrought iron, one inch in diameter—one on each side of the girder attached at the points *BB* to iron pins two inches in diameter, which passed under the bottom flange; at the points *AA* they were connected by means of plates clipping the ends of the girder, and tightened up by nuts and screws.

"A weight of 8,960 lbs. was placed upon the centre of the girder, and the deflection ($2\frac{1}{8}$") carefully taken.

"The tension rods were then put on and applied in various places, and the weight required to produce the same deflection is given in the following table; the distance of the bearings was in all cases 27 feet:—

No. of experiment.	Distance of points *A A above* the bottom of girder, in inch's	Distance of points *B B* on centre of pins, *below* the bottom of girders, in inc's	Weight required to produce the same deflection of 2⅛ inches, in lbs.
1	30	1	8848.
2	24	1	8960.
3	18	1	9072.
4	10	1	10192.
5	24	6¼	9856.
6	10	6¼	13552.
7	5	6¼	12544.

"After the above experiment had been made, the tension rods were removed, and the deflection with 8,960 lbs. was found to be the same as at the commencement.

"Our readers will observe that, strange as the fact may appear, when one end of each tension rod was 30 inches above the bottom of the girder, and the other end one inch below the bottom, the same deflection was produced by 112 lbs. less than when there were *no rods*.

When the distance of the points *AA* above the bottom of the girder was lessened to 10 inches, and the distance of the points of suspension (*BB*) *below* the bottom of the girder was increased to $6\frac{1}{4}$ inches, the girder was strengthened 50 per cent."

DEFLECTION OF BEAMS.

We give below the general formulas for deflections, from Prof. Moseley's "Mechanics of Engineering," edited by Prof. Mahan, 1856, merely changing the letters which he has employed, *to agree with those heretofore used.*

Let L=The total length of unsupported part of beam, or the clear span in inches.

Let I=The moment of inertia of the cross section, in respect to an axis passing through its centre of gravity (dimensions in inches).

Let E=The modulus of elasticity of the given material in lbs.

Let e=Deflection in inches.

Let W=The weight which produces the deflection (in lbs.).

Then, for a beam fixed at one end and loaded at the other,

$$e=\frac{WL^3}{3EI}, \quad \text{No. 33}$$

For a beam fixed at one end and uniformly loaded,

$$e=\frac{WL^3}{8EI}, \quad \text{No. 34}$$

For a beam supported at both ends and loaded at the centre,

$$e=\frac{WL^3}{48EI}, \quad \text{No. 35}$$

For a beam supported at both ends and uniformly loaded,

$$e=\frac{5}{8}\times\frac{WL^3}{48EI}, \quad \text{No. 36}$$

For rectangular sections, $I=\frac{1}{12}bd^3$, when b=breadth, and d=depth of beam. For **I** shaped and hollow rectangular sections, let b and d remain as before, and let b_1 and d_1 represent the breadth and the depth of hollow part; then

$I=\frac{1}{12}(bd^3—b_1d_1^3)$. For circular sections, let R be the radius; then $I=\frac{1}{4}—R^4=.7854\ R^4\ (R^4—r^4)$.

For annular sections or hollow cylinders, let R=outer radius, and r=inner radius; then $I=.7854\ R^4$.

The general law of deflections, as embodied in the foregoing formulas, and verified by the experiments of Prof. Barlow and others, is that they vary directly as the weight and cube of the length, and inversely as the breadth and cube of the depth, or as the moment of inertia of the cross section.

The deflections of a beam are always proportional to the load within the limit of elasticity; beyond that limit they increase rapidly, but do not seem to follow any general law.

Deflections of Rectangular Beams.

Let b=The breadth in inches
Let d= " depth "
Let e= " deflection "
Let W=The weight in lbs.
} As before.

Let L=The length of span in *feet*.

Let C=A co-efficient, according to the nature of the material.

Then, for a beam fixed at one end and loaded at the other,

$$e=C\frac{WL^3}{bd^3}, \quad \text{No. 37}$$

Beam fixed at one end and uniformly loaded,

$$e=\frac{3}{8}C\frac{WL^3}{bd^3}, \quad \text{No. 38}$$

Beam supported at both ends and loaded at the centre,

$$e=\frac{1}{16}C\frac{WL^3}{bd^3}, \quad \text{No. 39}$$

Beam supported at both ends and uniformly loaded,

$$e = \tfrac{5}{8} \bowtie \tfrac{1}{16} C \frac{WL^3}{bd^3}, \quad \text{No. 40}$$

As the majority of experiments which have been published were made on beams supported at both ends and loaded at the centre, we have deduced the values of C, as given below, from the formula $C = \frac{16ebd^3}{WL^3}$.

The deflection of a round beam, *whose diameter is equal to the side of the square beam*, is 1.7 times that of the square beam.

Values of C.

For wrought iron,	$C =$	From .0002 to .0003
For cast iron,	$C =$	From .00037 to .0004

TIMBER.	C
Ash,	.004
Beech,	.0048
Elm,	.0048 to .0084
Maple, common,	.0076
Willow,	.0124
Horse chestnut,	.0064
White pine, American,	.0048
Pitch pine, "	.0064
English oak,	.0044
Mahogany, Honduras,	.004
" Spanish,	.0052
Walnut, green,	.008
Cherry, "	.0052
Pear, "	.0084

TORSION.

The resistance of solid cylindrical shafts to a twisting strain increases directly as the cube of the diameter, while the strain is increased directly as the weight ⋈ the distance from the axis of shaft at which it acts.

Hence, representing by R the distance of the weight or pressure from the axis of shaft in feet, by D the diameter of the shaft in inches (see Fig. 22), by S the co-efficient of the given material, and by W the breaking pressure or weight in lbs., we have—

$$W = S\frac{D^3}{R}, \quad \text{No. 41}$$

$$\text{and } D = \sqrt[3]{\frac{WR}{S}} \quad \text{No. 42.}$$

When $D=1$ and $R=1$, then $\frac{D^3}{R}=1$, S then becomes equal to W—that is to say, S represents the breaking weight of a shaft one inch in diameter, applied at the end of a lever one foot long.

Torsional Strength of Hollow Shafts.

Let W, R, D, and S, be as above, and let d represent the interior diameter of the cylinder in inches, then—

$$W = S\frac{D^4 - d^4}{Dr}, \quad \text{No. 43}$$

Which is a practical formula deduced from Major Wade s experiments, by Lieut. Rodman, of the U. S. Ordnance department.

Values of S.

For wrought iron, $S=640$
" cast " $S=560$
" bronze " $S=460$

} Co-efficients for the *breaking* weights.

Relative torsional strength of cast iron shafts of different forms, havidg equal areas of cross sections.

From Maj. Wade's experiments on shafts whose cross sections were 1, 2 and 3 square inches.

Solid Cylinder	Solid Square.	Hollow cylinders whose interior and exterior diameters are in the proportion of				
		4 to 10	5 to 10	6 to 10	7 to 10	8 to 10
1.0000	0.8750	1.2656	1.4433	1.7000	2.0864	2.7377

The following Table exhibits the results of experiments made by Mr. Dunlop at Glasgow, on round bars of wrought iron.

Length of bars in inches.	Diameter of bars in inches.	Breaking weight in lbs.	Length of lever.	Values of $S = \frac{RW}{D^3}$
2¾	2	250	Lever 14 feet 2 in. long.	442
2¼	2¼	384		476
3	2½	408		370
3	2¾	700		476
4	3¼	1170		483
5	3½	1240		410
5	3¾	1662		446
5	4	1938		428
6	4¼	2158		398

Table showing the WORKING *Strength of Wrought Iron Shafts; the 2d. 4th. 6th and 8th. columns containing the products of the cube of the diam. in inches, by the coefficient of* SAFE WEIGHT.

Diam. of shaft in inches.	$D^3 \times 125$	Diam. of shaft in inches.	$D^3 \times 125$	Diam. of shaft in inches.	$D^3 \times 125$	Diam. of shaft in inches.	$D^3 \times 125$
¼	1 95	4¼	9587.	8¾	83625	13¼	290750.
⅜	6.58	4½	11387.	9	91125.	13½	307500.
½	15.6	4¾	13375.	9¼	98875.	13¾	325875
⅝	30 5	5	15625.	9½	107125	14	343000.
¾	52.5	5¼	18000.	9¾	115750.	14¼	361625.
1	125.	5½	20750.	10	125000.	14½	381000·
1¼	243.	5¾	23750.	10¼	134500.	14¾	401125.
1½	421.	6	27000.	10½	144625	15	420875.
1¾	668.	6¼	30500.	10¾	155250.	15¼	443250.
2	1000.	6½	34250.	11	166375	15½	465375.
2¼	1423.	6¾	38375	11¼	177875	16	512000.
2½	1950.	7	42875.	11½	190000	16½	551500·
2¾	2598	7¼	47625	11¾	202750.	17	614125.
3	3375.	7½	52625.	12	216000.	17½	669875.
3¼	4290.	7¾	58125.	12¼	229750	18	729000.
3½	5350.	8	64000	12½	244125	18½	791375.
3¾	6587.	8¼	70125.	12¾	259000.	19	857375
4	8000.	8½	76750	13	274625	20	1000000.

Example illustrating the use of the above table.

The pressure on the piston of a steam engine is 23805 lbs. and the length of crank 4 feet 6 inches. What diameter of shaft is necessary, if of wrought iron? Multiplying 23805 ⋈ 4 5 we obtain 107122, and looking in the table we find the diam. of shaft corresponding to it to be $9\frac{1}{2}$ inches. For from formula No. 41, we have

$$W = \frac{\text{co-efficient} \bowtie D^3}{R}, \text{ whence } WR = \text{co-efficient} \bowtie D^3.$$

That is, the product of the weight or pressure by the length of lever must be equal to the product of the cube of the diameter of shaft by the co-efficient.

Cast Iron Shafts.

It is customary, owing to the defective elasticity of cast iron, to make the diameter of cast iron shafts $1\frac{1}{10}$ times that diameter which would be proper if wrought iron were used.

Elasticity of Torsion.

Let D=The diameter of shaft in inches.

Let R=The length of crank or lever in inches.

Let L=The length of shaft in inches.

Let W=The weight at the end of the lever or crank in lbs.

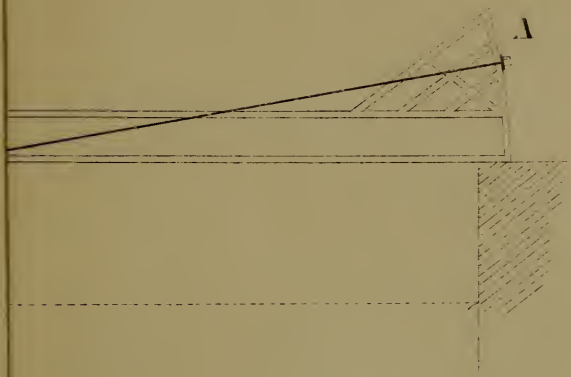
Θ=The angle in degrees through which the shaft is twisted; that is, supposing one end of the shaft fixed, Θ represents the number of degrees through which the opposite or free end is moved. If the shaft were twisted a complete revolution to the place whence it started, Θ would then be 360 degrees.

For wrought iron, $\Theta = \frac{WRL}{17500\ D^4}$, - - - No. 44

For cast iron, $\Theta = \frac{WRL}{10000\ D^4}$, - - - - No. 45

And for wood, $\Theta = \frac{WRL}{218\ D^4}$, - - - - No. 46

18 April 1860



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ILLUSTRATIONS OF THE STRENGTH OF MATERIALS.

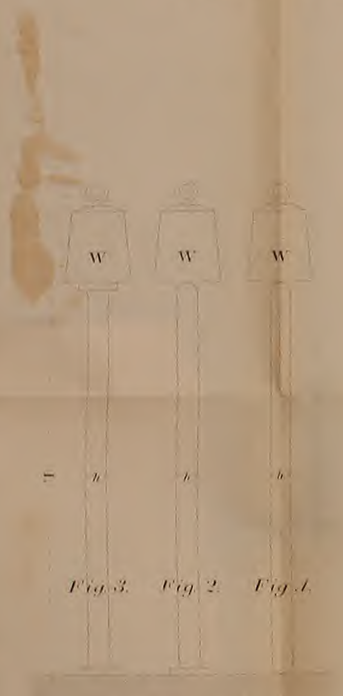


Fig 3. Fig 2. Fig 1.

Pillars

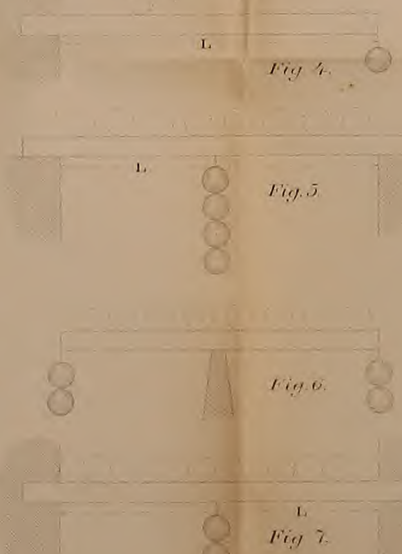


Fig 4.

Fig 5.

Fig 6.

Fig 7.

Fig 22.

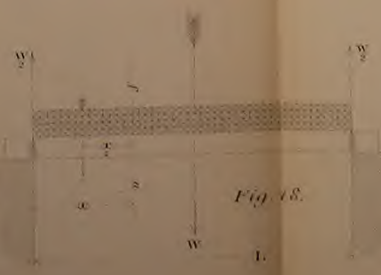


Fig 18.

Load at the free end

Fig 8. Fig 9. Fig 10.

Beams fixed at one end

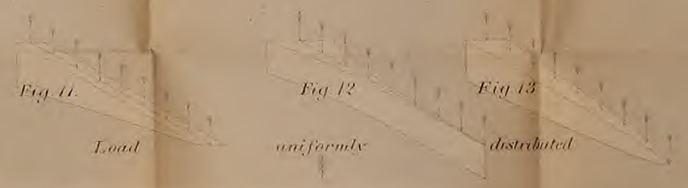


Fig 11. Fig 12. Fig 13.

Load uniformly distributed

Beams supported at both ends

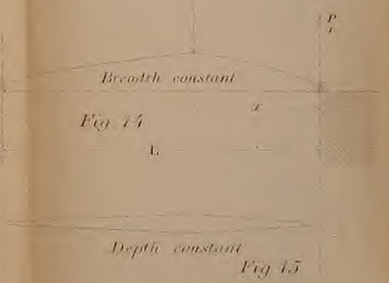


Fig 14

Depth constant

Fig 15

Load at any point between the supports

Fig 16.

Depth constant

Fig 17.

Load uniformly distributed

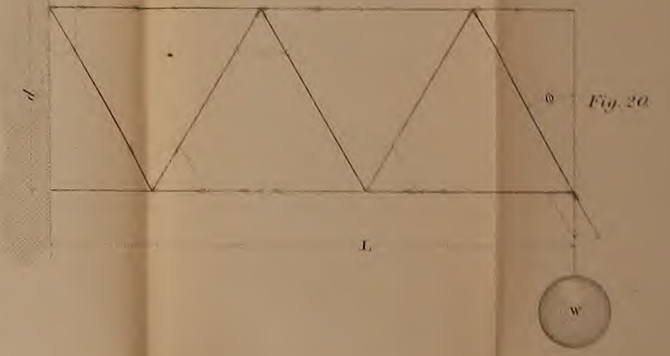
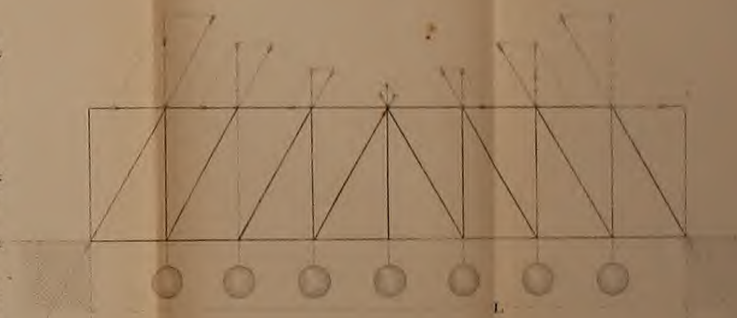


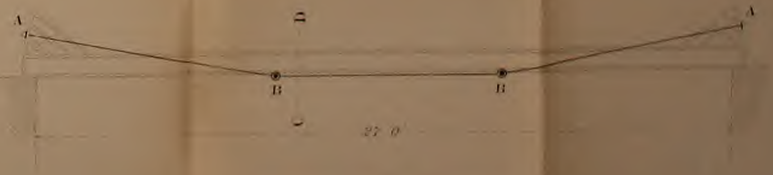
Fig 20.

Lattice Girders



Beams of uniform strength

Fig 21.



Trussed cast iron girders.

Fig 19

Enlarged section through C D

J R Whildin Civil Engineer

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